1. Raziskovalna organizacija (*Research organisation)*:

Univerza v Ljubljani, *Fakulteta za matematiko in fiziko*

2. Ime in priimek mentorja (*Name and surname of a mentor*):

Oliver Dragičević

3. Področje znanosti iz šifranta ARRS (*Primary research field*):

1.01.01 - Naravoslovno-matematične vede / Matematika / Analiza

4. Kontaktni e-naslov mentorja (*Contact of a mentor)*:

oliver.dragicevic@fmf.uni-lj.si

5. Kratek opis programa usposabljanja (*Short description of the program)*:

Kandidat se bo usposabljal na področju kompleksne analize v povezavi s harmonično analizo in teorijo minimalnih ploskev. Navedenih je nekaj primernih problemov za izdelavo disertacije.

V članku T. Iwaniec, G. Martin: Riesz Transforms and Related Singular Integrals, J. Reine Angew. Math. 473 sta avtorja obravnavala singularne integrale, ki se pojavijo v teoriji regularnosti nelinearnih PDE v poljubnih dimenzijah. Eden poudarkov v njunem delu je redukcija ocen operatorjev na R^n z vrednostmi v večdimenzionalnih prostorih na skalarne operatorje na R^2 oz. C. V prvi vrsti obravnavata L^p norme celih potenc t.i. *kompleksne Rieszove transformacije R*, znane tudi kot *kompleksna Hilbertova transformacija*, ki je definirana kot $R = R_2 + i\, R_1$. Ta operator je tesno povezan s kompleksno analizo, saj je R^2 t.i. Ahlfors-Beurlingov operator T, ki je opredeljen s tem, da slika $(\partial_x+i\partial_y)f$ v $(\partial_x-i\partial_y)f$, torej predstavlja most med temeljnima diferencialnima operatorjema prvega reda v kompleksni analizi. Z normami operatorja R^k na L^p, ki jih označimo s $H_p(k)$, so se nadalje v svoji monografiji ukvarjali Astala, Iwaniec in Saksman (Elliptic Partial Differential Equations and Quasiconformal Mappings in the Plane, Princeton Mathematical Series 48, Princeton University Press, 2009, Sec. 4.5), ki so dokazali, da $H_p(k)\leq C(p)(1+k^2)$. Pred nekaj leti je predlagatelj to oceno izboljšal na $H_p(k)\leq C\,|k|^{1-2/p}(p-1)$ za $p\geq 2$. Za sode k je to hkrati tudi spodnja ocena za $H_p(k)$, kot je dokazal O. Dragičević (Some remarks on the L^p estimates for powers of the Ahlfors-Beurling operator, Arch. Math. 96 (2011), 463-47). S tem dobimo dvostransko obnašanje L^p norm celih potenc operatorja T (t.j., *sodih* potenc R). Predlagani problem je najti natančne dvostranske ocene za L^p norme *vseh* celih potenc operatorja R. Očitno je $R^{2l+1}=RT^l$. Toda R ni diferencialni operator, kar terja nov pristop k iskanju ekstremalne družine funkcij v primeru lihih k. Rešitev problema bi predstavljala nov prispevek k razumevanju povezave med singularnimi integrali, kompleksno analizo ter PDE.

Drug povezan sklop so problemi moderne teorije Oka. Pred kratkim sta F. Forstnerič in F.Larusson raziskala lastnosti Oka grupe holomorfnih avtomorfizmov prostora C^n (Math. Res. Lett. 21 (2014) 1047-1067). Odprt problem je raziskati te lastnosti za druge Steinove mnogoterosti z Varolinovo lastnostjo gostote (kompletna holomorfna vektorska polja gosto generirajo algebro vseh vektorskih polj). Trenutno se v svetu intenzivno študira razrede afinih kompleksnih mnogoterosti z lastnostjo gostote ali s holomorfno fleksibilnostjo; obe lastnosti zagotavljata lastnost Oka. Na tem področju je vrsta primernih odprtih problemov, kjer se da uporabljati nove metode F. Kutzschebaucha in S.

Kalimana (On algebraic volume density property, Transform. Groups 21 (2016), 2, 451-478; On the density and volume density property, Complex analysis and geometry, 175-186, Springer Proc. Math. Stat, 144, Springer, Tokyo, 2015).

V delu F. Forstnerič, F. Larusson, Holomorphic flexibility properties of compact complex surfaces, Int. Math. Res. Notices (2014) 13, 3714-3734, sta avtorja poleg novih rezultatov prikazala pregled znanih lastnostih Oka v razredu kompaktnih kompleksnih ploskev nesplošnega tipa. Največji večinoma neraziskan razred ostajajo K3 ploskve in še posebej eliptične ploskve. Na tem področju je vrsta primernih problemov za izdelavo disertacije. V povezavi s teorijo minimalnih ploskev in PDE je aktualen problem lastnosti Oka kompleksnih integralnih krivulj diferencialnih sistemov; gre za povezavo s h-principom za usmerjene imerzije (M. Gromov, Partial Differential Relations, Springer, Berlin 1986). Pomeben korak v tej smeri so bili nedavni članki F. Forstneriča s sodelavci (F.Forstnerič, A. Alarcon, Null curves and directed immersions of open Riemann surfaces, Invent. Math. 196 (2014) 733-771; A. Alarcon, B.Drinovec Drnovšek, F. Forstnerič, F.J.Lopez, Every bordered Riemann surface is a complete conformal minimal surface bounded by Jordan curves. Proc. London Math. Soc. (3) 111 (2015) 851-886).

In 1996 Iwaniec and Martin studied singular integrals that appear in regularity theory of nonlinear PDE in arbitrary dimensions (Riesz Transforms and Related Singular Integrals, J. Reine Angew. Math. 473). One of the key features of their work is that they reduce the estimates of vector-valued operators on R^n (such as combinations of Riesz transforms, complex Riesz transforms, certain differential operators, etc.) to those of scalar-valued operators on R^2, *i.e. C*. Most prominently, they study the L^p norms of integer powers of the so-called *complex Riesz transform R*, also called the *complex Hilbert transform*, which is defined as $R = R_2 + i R_1$. This operator is closely related to complex analysis, since R^2 is the Ahlfors-Beurling operator *T* which is characterized by mapping $(\partial_x+i\partial_y)f$ into $(\partial_x-i\partial_y)f$, i.e., by intertwining two fundamental first-order differential operators in complex analysis. The L^p norms of R^k, denoted by $H_p(k)$, were further treated in the monograph by Astala, Iwaniec and Saksman (Elliptic Partial Differential Equations and Quasiconformal Mappings in the Plane, Princeton Mathematical Series 48, Princeton University Press, 2009, Section 4.5], where they proved that $H_p(k) \le C(p)(1+k^2)$. A few years ago, O. Dragičević improved this estimate to $H_p(k) \le C\,|k|^{1-2/p}(p-1)$ for $p \ge 2$ (unpublished manuscript). In case of even *k*, this is also a lower estimate for $H_p(k)$, as proven by Dragičević (Some remarks on the L^p estimates for powers of the Ahlfors-Beurling operator, Arch. Math. 96 (2011), 463-471). Hence this gives the sharp two-sided behaviour of the L^p norms of integer powers of *T* (i.e., of *even* integer powers of *R*). The proposed problem would be to obtain sharp two-sided behaviour of the L^p norms of *all* integer powers of *R*. Clearly $R^{2l+1}=RT^l$. However, *R* is not a differential operators, thus a new approach to obtaining extremal family of functions would be needed in the case of odd *k*. A solution of this problem would represent a new contribution to understanding the interplay of singular integrals, complex analysis and PDE.

Another related group of suitable PhD problems arises from modern Oka theory. Recently, Forstnerič and Larusson studied Oka properties of the group of all holomorphic automorphisms of complex Euclidean spaces C^n for n>1 (F.Forstnerič, F.Larusson, Math. Res. Lett. 21 (2014), 1047-1067). An important open problem is to understand whether any or all of these properties hold for other Stein manifolds with Varolin's density property (i.e., complete holomorphic vector fields densely generate the algebra of all holomorphic vector fields on the manifold). This topic is closely related to the Andersen-Lempert theory which is very useful in holomorphic dynamics and elsewhere in complex analysis. Validity of the density property, and of the related holomorphic flexibility property, in the class of affine complex manifolds is a subject of intensive contemporary investigation by several groups worldwide. This topic offers a variety of suitable PhD problems where one could use the newly developed methods of F. Kutzschebauch and S. Kalimana (On algebraic volume density property, Transform. Groups 21 (2016), 2, 451-478; On the density and volume density property, Complex analysis and geometry, 175-186, Springer Proc. Math. Stat, 144, Springer, Tokyo, 2015).

In the paper F. Forstnerič, F. Larusson, Holomorphic flexibility properties of compact complex surfaces, Int. Math. Res. Notices (2014) 13, 3714-3734, the authors summarized the state of the art concerning the present knowledge of the Oka properties in the class of compact complex surfaces of nongeneral type. The most important class which remains poorly understood are the K3 surfaces, and in particular the elliptic surfaces. This area offers suitable problems for PhD dissertation. This subject is furthermore related to the theory of minimalnih surfaces, the common denominator being the study of Oka properties of holomorphic integral curves in exterior differential systems. This forms a link with the h-principle for directed immersions (M.Gromov, Partial Differential Relations, Springer, Berlin 1986). An important step in this direction has been made recently by F.Forstnerič and collaborators (see for example F.Forstnerič, A.Alarcon, Null curves and directed immersions of open Riemann surfaces, Invent. Math. 196 (2014) 733-771; also A. Alarcon, B.Drinovec Drnovšek, F. Forstnerič, F.J.Lopez, Every bordered Riemann surface is a complete conformal minimal surface bounded by Jordan curves. Proc. London Math. Soc. (3) 111 (2015) 851-886).